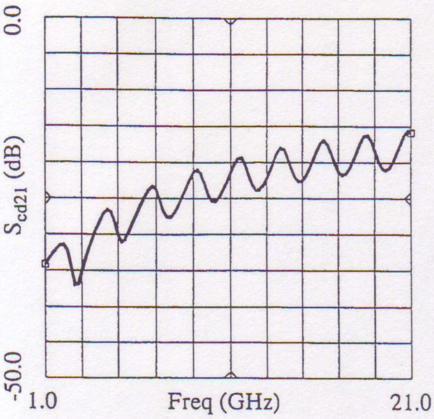

Fig. 9. Simulated magnitude in dB of S_{cd21} versus frequency for asymmetric coupled-pair line.

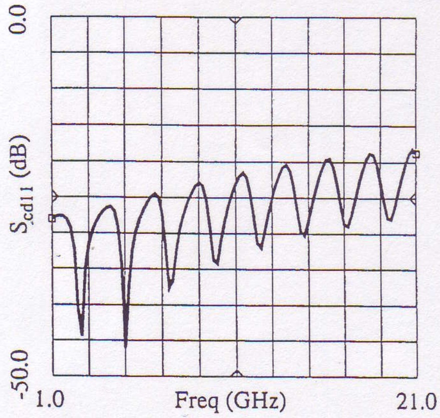

Fig. 10. Simulated magnitude in dB of S_{cd11} versus frequency for asymmetric coupled-pair line.

the differential-mode transmission S_{dd21} correspond to a worst case point in the relative phases of S_{dd21}, S_{cd21}, and S_{cd11}. In a low loss transmission line case, the insertion loss due to mode conversion and miss-match can be shown to be approximately

$$\text{Loss (dB)} \approx -10\log[1 - (|S_{dd11}|^2 + |S_{cd21}|^2 + |S_{cd11}|^2)]. \tag{29}$$

This is consistent with the increasing ripple in S_{dd21} with increasing frequency since the mode conversion (S_{cd21} and S_{cd11}) increases with frequency.

VI. Conclusion

A theory for mixed-mode s-parameters is developed for characterization of microwave differential circuits. The theory is based on microwave coupled line systems, and is useful to describe general differential circuits, including coupled transmission lines. The theory is applied to develop the concept of an ideal mixed-mode s-parameter measurement system, the restriction of equal even and odd-mode characteristic impedances is shown to result in useful relationships for such a system. A real mixed-mode measurement system can be implemented from the results of this theoretical work. However, a proper mathematical basis is needed in the future for characterization and calibration of these measurements. Finally, microwave simulations illustrate some of the utility of mixed-mode s-parameters.

Appendix
Transmission of Modes from Coupled to Uncoupled Lines

Consider a system where a pair of coupled transmission lines are connected in cascade with a pair of uncoupled transmission lines, as shown in Fig. A-1. The coupled pair will be considered to be a reference line as defined in Section III; hence, the coupled pair line is symmetric and low loss. The normalized waves at the outputs of the uncoupled lines will be investigated under the same assumptions, namely low loss and symmetry, which for the uncoupled case means the lines are identical. The voltages at a point x on the coupled pair lines are given by (12), rewritten hear to explicitly show the complex exponentials

$$\begin{aligned} v_1(x) &= V_e^{\text{pos}} e^{-\gamma_e x} + V_e^{\text{neg}} e^{\gamma_e x} + V_o^{\text{pos}} e^{-\gamma_o x} + V_o^{\text{neg}} e^{\gamma_o x} \\ v_2(x) &= V_e^{\text{pos}} e^{-\gamma_e x} + V_e^{\text{neg}} e^{\gamma_e x} - V_o^{\text{pos}} e^{-\gamma_o x} - V_o^{\text{neg}} e^{\gamma_o x} \end{aligned} \tag{A-1}$$

and the currents, also given by (12) are

$$\begin{aligned} i_1(x) &= \frac{V_e^{\text{pos}}}{Z_e} e^{-\gamma_e x} - \frac{V_e^{\text{neg}}}{Z_e} e^{\gamma_e x} + \frac{V_o^{\text{pos}}}{Z_o} e^{-\gamma_o x} - \frac{V_o^{\text{neg}}}{Z_o} e^{\gamma_o x} \\ i_2(x) &= \frac{V_e^{\text{pos}}}{Z_e} e^{-\gamma_e x} - \frac{V_e^{\text{neg}}}{Z_e} e^{\gamma_e x} - \frac{V_o^{\text{pos}}}{Z_o} e^{-\gamma_o x} + \frac{V_o^{\text{neg}}}{Z_o} e^{\gamma_o x}. \end{aligned} \tag{A-2}$$

With the uncoupled transmission lines, the voltages and currents at a point x are

$$\begin{aligned} v_{ui}(x) &= V_{ui}^{\text{pos}} e^{-\gamma_u x} + V_{ui}^{\text{neg}} e^{\gamma_u x} \\ i_{ui}(x) &= \frac{V_{ui}^{\text{pos}}}{Z_u} e^{-\gamma_u x} - \frac{V_{ui}^{\text{neg}}}{Z_u} e^{\gamma_u x} \end{aligned} \tag{A-3}$$

with $i = 1, 2$ and $Z_{u1} = Z_{u2} = Z_u, \gamma_{u1} = \gamma_{u2} = \gamma_u$. At the interface between the coupled pair and the uncoupled pair, $(x = 0, x' = d)$ the voltages and currents of the two systems must conform to the boundary conditions

$$\begin{aligned} v_{u1}(0) &= v_1(0) \quad i_{u1}(0) = i_1(0) \\ v_{u2}(0) &= v_2(0) \quad i_{u2}(0) = i_2(0). \end{aligned} \tag{A-4}$$

Through the application of these boundary conditions and (A-1)–(A-3), the phasor coefficients on the uncoupled lines are found to be

$$\begin{aligned} V_{u1}^{\text{pos}} &= \frac{1}{2}\Big[V_e^{\text{pos}}\Big(1 + \frac{Z_u}{Z_e}\Big) + V_e^{\text{neg}}\Big(1 - \frac{Z_u}{Z_e}\Big) \\ &\quad + V_o^{\text{pos}}\Big(1 + \frac{Z_u}{Z_o}\Big) + V_o^{\text{neg}}\Big(1 - \frac{Z_u}{Z_o}\Big)\Big] \\ V_{u1}^{\text{neg}} &= \frac{1}{2}\Big[V_e^{\text{pos}}\Big(1 - \frac{Z_u}{Z_e}\Big) + V_e^{\text{neg}}\Big(1 + \frac{Z_u}{Z_e}\Big) \\ &\quad + V_o^{\text{pos}}\Big(1 - \frac{Z_u}{Z_o}\Big) + V_o^{\text{neg}}\Big(1 + \frac{Z_u}{Z_o}\Big)\Big] \end{aligned} \tag{A-5}$$