These relations between the even/odd mode characteristic impedances and the differential/common mode characteristic impedances are consistent with the matched load terminations discussed in the literature [7], [8].

Now that voltages, currents, and characteristic impedances have been defined for both differential and common modes, the normalized power waves can be developed. By the definition for a generalized power wave at the nth port [23], [24]

$$a_n = \frac{1}{2\sqrt{\mathrm{Re}(Z_n)}}[v_n + i_n Z_n]$$
$$b_n = \frac{1}{2\sqrt{\mathrm{Re}(Z_n)}}[v_n - i_n Z_n^*] \tag{16}$$

where a_n is the normalized wave propagating in the forward (positive x) direction, b_n is the normalized wave propagating in the reverse (negative x) direction, and Z_n is the characteristic impedance of the port. With the above definitions, the differential normalized waves become, at port 1

$$a_{\mathrm{dm}_1} \equiv a_{\mathrm{dm}}(0) = \frac{1}{2\sqrt{\mathrm{Re}(Z_{\mathrm{dm}})}}[v_{\mathrm{dm}}(x) + i_{\mathrm{dm}}(x)Z_{\mathrm{dm}}]|_{x=0}$$
$$b_{\mathrm{dm}_1} \equiv b_{\mathrm{dm}}(0) = \frac{1}{2\sqrt{\mathrm{Re}(Z_{\mathrm{dm}})}}[v_{\mathrm{dm}}(x) - i_{\mathrm{dm}}(x)Z_{\mathrm{dm}}^*]|_{x=0}. \tag{17}$$

Similarly, define the common-mode normalized waves, at port 1, as

$$a_{\mathrm{cm}_1} \equiv a_{\mathrm{cm}}(0) = \frac{1}{2\sqrt{\mathrm{Re}(Z_{\mathrm{cm}})}}[v_{\mathrm{cm}}(x) + i_{\mathrm{cm}}(x)Z_{\mathrm{cm}}]|_{x=0}$$
$$b_{\mathrm{cm}_1} \equiv b_{\mathrm{cm}}(0) = \frac{1}{2\sqrt{\mathrm{Re}(Z_{\mathrm{cm}})}}[v_{\mathrm{cm}}(x) - i_{\mathrm{cm}}(x)Z_{\mathrm{cm}}^*]|_{x=0}. \tag{18}$$

Analogous definitions at port 2 can easily be found by setting $x = l$.

Imposing the condition of low-loss transmission lines on the coupled-pair of Fig. 2, the characteristic impedances are approximately purely real [24]. Under this restriction, $Z_{\mathrm{dm}} \approx \mathrm{Re}\{Z_{\mathrm{dm}}\} \equiv R_{\mathrm{dm}}$ and $Z_{\mathrm{cm}} \approx \mathrm{Re}\{Z_{\mathrm{cm}}\} \equiv R_{\mathrm{cm}}$. With this assumption, the normalized wave equations at port 1 can be simplified

$$a_{\mathrm{dm}_1} = \frac{1}{2\sqrt{R_{\mathrm{dm}}}}[v_{\mathrm{dm}}(x) + i_{\mathrm{dm}}(x)R_{\mathrm{dm}}]|_{x=0}$$
$$b_{\mathrm{dm}_1} = \frac{1}{2\sqrt{R_{\mathrm{dm}}}}[v_{\mathrm{dm}}(x) - i_{\mathrm{dm}}(x)R_{\mathrm{dm}}]|_{x=0} \tag{19}$$
$$a_{\mathrm{cm}_1} = \frac{1}{2\sqrt{R_{\mathrm{cm}}}}[v_{\mathrm{cm}}(x) + i_{\mathrm{cm}}(x)R_{\mathrm{cm}}]|_{x=0}$$
$$b_{\mathrm{cm}_1} = \frac{1}{2\sqrt{R_{\mathrm{cm}}}}[v_{\mathrm{cm}}(x) - i_{\mathrm{cm}}(x)R_{\mathrm{cm}}]|_{x=0}. \tag{20}$$

With the normalized power waves defined, the development of mixed-mode s-parameters is straight forward. The definition of generalized s-parameters [23], [24] is

$$[b] = [S][a] \tag{21}$$

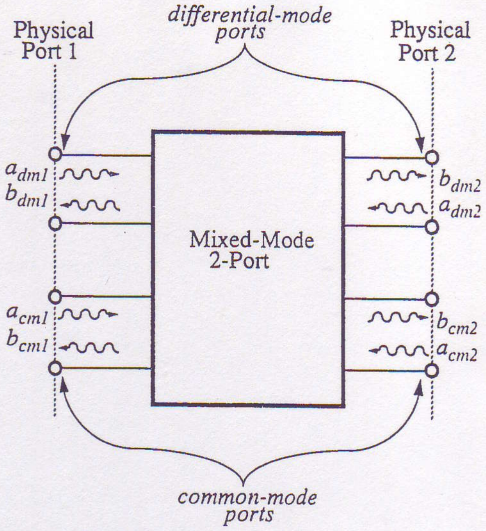

Fig. 3. Conceptual diagram of mixed-mode two-port.

where the bold letters denote an n-dimensional column vector or an n-by-n matrix. Given a coupled-line two-port like Fig. 2, or any arbitrary mixed-mode two-port, the generalized mixed-mode s-parameters can be given as

$$\begin{aligned}
b_{\mathrm{dm}_1} &= s_{11}a_{\mathrm{dm}_1} + s_{12}a_{\mathrm{dm}_2} + s_{13}a_{\mathrm{cm}_1} + s_{14}a_{\mathrm{cm}_2}\\
b_{\mathrm{dm}_2} &= s_{21}a_{\mathrm{dm}_1} + s_{22}a_{\mathrm{dm}_2} + s_{23}a_{\mathrm{cm}_1} + s_{24}a_{\mathrm{cm}_2}\\
b_{\mathrm{cm}_1} &= s_{31}a_{\mathrm{dm}_1} + s_{32}a_{\mathrm{dm}_2} + s_{33}a_{\mathrm{cm}_1} + s_{34}a_{\mathrm{cm}_2}\\
b_{\mathrm{cm}_2} &= s_{41}a_{\mathrm{dm}_1} + s_{42}a_{\mathrm{dm}_2} + s_{43}a_{\mathrm{cm}_1} + s_{44}a_{\mathrm{cm}_2}
\end{aligned} \tag{22}$$

where the subscripts 1 and 2 denote ports 1 and 2, respectively. Here, $[S]$ can be described by

$$\begin{bmatrix} b_{\mathrm{dm}_1} \\ b_{\mathrm{dm}_2} \\ b_{\mathrm{cm}_1} \\ b_{\mathrm{cm}_2} \end{bmatrix} = \left[\begin{array}{c|c} S_{dd} & S_{dc} \\ \hline S_{cd} & S_{cc} \end{array}\right] \begin{bmatrix} a_{\mathrm{dm}_1} \\ a_{\mathrm{dm}_2} \\ a_{\mathrm{cm}_1} \\ a_{\mathrm{cm}_2} \end{bmatrix}. \tag{23}$$

The following names are used: Call $[S_{dd}]$ the differential s-parameters, $[S_{cc}]$ the common-mode s-parameters, and $[S_{dc}]$ and $[S_{cd}]$ the mode-conversion or cross-mode s-parameters. In particular, $[S_{dc}]$ describes the conversion of common-mode waves into differential-mode waves, and $[S_{cd}]$ describes the conversion of differential-mode waves into common-mode waves. These four partitions are analogues to four transfer gains $(A_{cc}, A_{dd}, A_{cd}, A_{dc})$ introduced by Middlebrook [2].

These mixed-mode two-port s-parameters can be shown graphically (see Fig. 3) as a traditional four-port. It must be remembered, however, that the ports are conceptual tools only, and not physically separate ports.

IV. Considerations for a Practical Mixed-Mode Measurement System

The most straightforward means of implementing a mixed-mode s-parameter measurement system is to directly apply differential and common-mode waves while measuring the resulting differential and common-mode waves. Unfor-