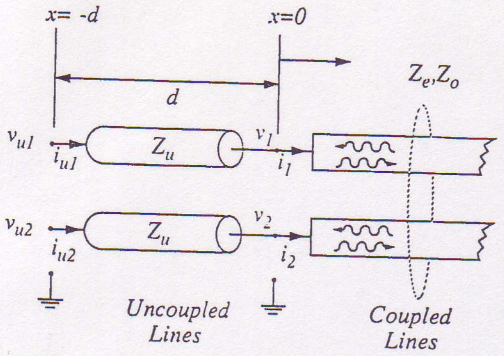

Fig. A-1. Schematic of uncoupled pair in cascade with coupled pair-line.

$$
\begin{aligned}
V_{u2}^{\text{pos}} &= \frac{1}{2}\left[V_e^{\text{pos}}\left(1+\frac{Z_u}{Z_e}\right) + V_e^{\text{neg}}\left(1-\frac{Z_u}{Z_e}\right)\right.\\
&\quad \left. - V_o^{\text{pos}}\left(1+\frac{Z_u}{Z_o}\right) - V_o^{\text{neg}}\left(1-\frac{Z_u}{Z_o}\right)\right]\\
V_{u2}^{\text{neg}} &= \frac{1}{2}\left[V_e^{\text{pos}}\left(1-\frac{Z_u}{Z_e}\right) + V_e^{\text{neg}}\left(1+\frac{Z_u}{Z_e}\right)\right.\\
&\quad \left. - V_o^{\text{pos}}\left(1-\frac{Z_u}{Z_o}\right) - V_o^{\text{neg}}\left(1+\frac{Z_u}{Z_o}\right)\right].
\end{aligned} \tag{A-6}
$$

The differential-mode voltage at the output of the uncoupled pair ($x = -d$) can be defined by (6) as

$$v_{\text{dm}_u}(-d) = v_{u1}(-d) - v_{u2}(-d) \tag{A-7}$$

which can be found to be

$$v_{\text{dm}_u}(-d) = V_{\text{dm}_u}^{\text{pos}} e^{\gamma_u d} + V_{\text{dm}_u}^{\text{neg}} e^{-\gamma_u d} \tag{A-8}$$

where

$$V_{\text{dm}_u}^{\text{pos}} = V_{u1}^{\text{pos}} - V_{u2}^{\text{pos}} \qquad V_{\text{dm}_u}^{\text{neg}} = V_{u1}^{\text{neg}} - V_{u2}^{\text{neg}}. \tag{A-9}$$

The normalized forward differential-mode wave at the output of the coupled pair, defined generally by (16), can be shown as

$$a_{\text{dm}_u} = \frac{V_{\text{dm}_u}^{\text{pos}}}{\sqrt{R_{\text{dm}_u}}} \tag{A-10}$$

where $R_{\text{dm}u}$ is the (approximately) purely real characteristic impedance of the differential-mode, defined between the uncoupled lines, and $R_{\text{dm}u} = 2R_u$ where R_u is the characteristic impedance of the each of the uncoupled lines. From (A-5), (A-6), (A-9) and (A-10), it is found that

$$
\begin{aligned}
a_{\text{dm}_u} &= \frac{1}{2}\sqrt{\frac{R_{\text{dm}}}{R_{\text{dm}_u}}}\left[a_{\text{dm}}\left(1+\frac{R_{\text{dm}_u}}{R_{\text{dm}}}\right)\right.\\
&\quad \left. + b_{\text{dm}}\left(1-\frac{R_{\text{dm}_u}}{R_{\text{dm}}}\right)\right] e^{j\beta_u d}
\end{aligned} \tag{A-11}
$$

where a_{dm} and b_{dm} is the differential-mode normalized forward and reverse waves of the coupled system at $x = 0$, and R_{dm} is the approximately real characteristic impedance of the differential-mode on the coupled-pair. Similarly, the remaining differential and common-mode normalized waves can be shown to be

$$
\begin{aligned}
b_{\text{dm}_u} &= \frac{1}{2}\sqrt{\frac{R_{\text{dm}}}{R_{\text{dm}_u}}}\left[b_{\text{dm}}\left(1+\frac{R_{\text{dm}_u}}{R_{\text{dm}}}\right)\right.\\
&\quad \left. + a_{\text{dm}}\left(1-\frac{R_{\text{dm}_u}}{R_{\text{dm}}}\right)\right] e^{-j\beta_u d}\\
a_{\text{cm}_u} &= \frac{1}{2}\sqrt{\frac{R_{\text{cm}}}{R_{\text{cm}_u}}}\left[a_{\text{cm}}\left(1+\frac{R_{\text{cm}_u}}{R_{\text{cm}}}\right)\right.\\
&\quad \left. + b_{\text{cm}}\left(1-\frac{R_{\text{cm}_u}}{R_{\text{cm}}}\right)\right] e^{j\beta_u d}\\
b_{\text{cm}_u} &= \frac{1}{2}\sqrt{\frac{R_{\text{cm}}}{R_{\text{cm}_u}}}\left[b_{\text{cm}}\left(1+\frac{R_{\text{cm}_u}}{R_{\text{cm}}}\right)\right.\\
&\quad \left. + a_{\text{cm}}\left(1-\frac{R_{\text{cm}_u}}{R_{\text{cm}}}\right)\right] e^{-j\beta_u d}
\end{aligned} \tag{A-12}
$$

where $b_{\text{dm}}, a_{\text{cm}}$, and b_{cm} are the normalized waves of the coupled system at $x = 0$. Therefore, the differential and common-mode normalized waves at the output of the uncoupled lines are equal to the corresponding coupled system waves with a phase-shift and a scaling factor due to the different characteristic impedances. To the resulting mixed-mode s-parameters, the phase-shift and the scaling factor represent an arbitrary reference plane shift and a re-normalization to the characteristic impedance of the uncoupled transmission lines, respectively. Because of this, the coupled pair reference line can be replaced with an uncoupled pair reference, and the resulting mixed-mode s-parameters are simply transposed to a different reference impedance by the uncoupled lines. Therefore, the mixed-mode s-parameters of an arbitrary n-port DUT can be measured with $n$pairs of uncoupled transmission lines.

REFERENCES

[1] P. R. Gray and R. G. Meyer, *Analysis and Design of Analog Integrated Circuits*, 3rd ed. New York: Wiley, 1993.

[2] R. D. Middlebrook, *Differential Amplifiers*. New York: Wiley, 1963.

[3] T. T. Ha, *Solid-State Microwave Amplifier Design*, reprint ed. Malabar, FL: Krieger, 1991.

[4] *Application Note: WPH-505/805 Waveguide-Input Wafer Probes*. Beaverton, OR: Cascade Microtech, 1993.

[5] D. E. Carlton *et. al.*, "Microwave wafer probing," *Microwave J.*, vol. 29, pp. 121–129, Jan. 1985.

[6] V. K. Tripathi, "Asymmetric coupled transmission lines in an inhomogeneous medium," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, pp. 734–739, Sept. 1975.

[7] K. D. Marx, "Propagation modes, equivalent circuits, and characteristic terminations for multiconductor transmission lines with inhomogeneous dielectrics," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-21, pp. 450–457, Nov. 1973.

[8] H. Amemiya, "Time-domain analysis of multiple parallel transmission lines," *RCA Rev.*, vol. 28, pp. 241–276, June 1967.

[9] R. A. Speciale and V. K. Tripathi, "Wave-modes and parameter matrices of non-symmetrical coupled lines in a non-homogeneous medium," *Int. J. Electron.*, vol. 40, pp. 371–375, Apr. 1976.

[10] G. I. Zysman and A. K. Johnson, "Coupled transmission line networks in an inhomogeneous dielectric medium," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-17, pp. 753–759, Oct. 1969.

[11] E. G. Vlostovskiy, "Theory of coupled transmission lines," *Telecommun. Radio Eng.*, vol. 21, pp. 87–93, Apr. 1967.

[12] S. B. Cohn, "Characteristic impedances of broadside-coupled strip transmission lines," *IRE Trans. Microwave Theory Tech.*, vol. MTT-10, pp. 633–637, Nov. 1960.