Each voltage/current pair at each node represent a single propagating signal referenced to the ground potential. These signals will be called nodal waves.

A practical simplification in the development of mixed-mode s-parameter theory is to assume symmetric coupled pairs (i.e., lines A and B have equal width) as reference transmission lines. This assumption allows simple mathematical formulations of mixed-mode s-parameters. Furthermore, this assumption is not overly limiting, since reference lines may be made arbitrarily short. For symmetrical lines, in (2) $R_c = 1$ and $R_\pi = -1$, and the c and the π-modes become the *even* and *odd* modes, respectively, as first used by Cohn [13]. For notational purposes, we shall use the substitutions $c \rightarrow e$ and $\pi \rightarrow o$ for even-mode and odd-mode, respectively. With these substitutions, the mode characteristic impedances and propagation constants become

$$\begin{aligned} Z_{c1} &= Z_{c2} = Z_e \\ Z_{\pi 1} &= Z_{\pi 2} = Z_o \\ \gamma_c &= \gamma_e \quad \gamma_\pi = \gamma_o. \end{aligned} \tag{4}$$

Expressing (2) in the symmetric case

$$\begin{aligned} v_1 &= A_1 e^{-\gamma_e x} + A_2 e^{\gamma_e x} + A_3 e^{\gamma_o x} + A_4 e^{\gamma_o x} \\ v_2 &= A_1 e^{-\gamma_e x} + A_2 e^{\gamma_e x} - A_3 e^{-\gamma_o x} - A_4 e^{\gamma_o x} \\ i_1 &= \frac{A_1}{Z_e} e^{-\gamma_e x} - \frac{A_2}{Z_e} e^{\gamma_e x} + \frac{A_3}{Z_o} e^{-\gamma_o x} - \frac{A_4}{Z_o} e^{\gamma_o x} \\ i_2 &= \frac{A_1}{Z_e} e^{-\gamma_e x} - \frac{A_2}{Z_e} e^{\gamma_e x} - \frac{A_3}{Z_o} e^{-\gamma_o x} + \frac{A_4}{Z_o} e^{\gamma_o x}. \end{aligned} \tag{5}$$

As before, these voltage/current pairs are nodal waves at each terminal that are referenced to ground.

It is important, now, to define the differential and common-mode voltages and currents to develop a self-consistent set of mixed-mode s-parameters. Define the differential-mode voltage at a point, x, to be the difference of between voltages on node 1 and node 2

$$v_{\mathrm{dm}}(x) \equiv v_1 - v_2. \tag{6}$$

This standard definition establishes a signal that is no longer referenced to ground. In a differential circuit, one would expect equal current magnitudes to enter the positive input terminal as leaves the negative input terminal. Therefore, the differential-mode current is defined as one-half the difference between currents entering nodes 1 and 2

$$i_{\mathrm{dm}}(x) \equiv \tfrac{1}{2}(i_1 - i_2). \tag{7}$$

Definitions in (6) and (7) are self-consistent with the differential power delivered to a differential load. These definitions differ from previously published definitions by Zysman and Johnson [10] due to change in references. The common-mode voltage in a differential circuit is typically the average voltage at a port. Hence, common-mode voltage is one half the sum of the voltages on nodes 1 and 2

$$v_{\mathrm{cm}}(x) \equiv \tfrac{1}{2}(v_1 + v_2). \tag{8}$$

The common-mode current at a port is simply the total current flowing into the port. Therefore, define the common-mode current as the sum of the currents entering nodes 1 and 2

$$i_{\mathrm{cm}}(x) \equiv i_1 + i_2. \tag{9}$$

Note: The return current for the common-mode signal flows through the ground plane. Again, these definitions differ from definitions from Zysman and Johnson [10] due to change in references.

Expressing these differential and common-mode values (6) through (9) in terms of the line voltages and currents (5)

$$\begin{aligned} v_{\mathrm{dm}}(x) &= 2(A_3 e^{-\gamma_o x} + A_4 e^{\gamma_o x}) \\ i_{\mathrm{dm}}(x) &= \frac{A_3}{Z_o} e^{-\gamma_o x} - \frac{A_4}{Z_o} e^{\gamma_o x} \\ v_{\mathrm{cm}}(x) &= A_1 e^{-\gamma_e x} + A_2 e^{\gamma_e x} \\ i_{\mathrm{cm}}(x) &= 2\left(\frac{A_1}{Z_e} e^{-\gamma_e x} - \frac{A_2}{Z_e} e^{\gamma_e x}\right). \end{aligned} \tag{10}$$

Recall that A_1 and A_2 are the forward and reverse phasor coefficient for the even-mode propagation, and A_3 and A_4 are the forward and reverse phasor coefficient for the odd-mode propagation. If a short hand notation is introduced, a better understanding of these definitions can be had. Let

$$\begin{aligned} v_o^{\mathrm{pos}}(x) &\equiv A_3 e^{-\gamma_o x} & v_o^{\mathrm{neg}}(x) &\equiv A_4 e^{\gamma_o x} \\ v_e^{\mathrm{pos}}(x) &\equiv A_1 e^{-\gamma_e x} & v_e^{\mathrm{neg}}(x) &\equiv A_2 e^{\gamma_e x} \\ i_o^{\mathrm{pos}}(x) &\equiv \frac{A_3}{Z_o} e^{-\gamma_o x} & i_o^{\mathrm{neg}}(x) &\equiv \frac{A_4}{Z_o} e^{\gamma_o x} \\ i_e^{\mathrm{pos}}(x) &\equiv \frac{A_1}{Z_e} e^{-\gamma_e x} & i_e^{\mathrm{neg}}(x) &\equiv \frac{A_2}{Z_e} e^{\gamma_e x}. \end{aligned} \tag{11}$$

Then (5) becomes

$$\begin{aligned} v_1 &= v_e^{\mathrm{pos}}(x) + v_e^{\mathrm{neg}}(x) + v_0^{\mathrm{pos}}(x) + v_o^{\mathrm{neg}}(x) \\ v_2 &= v_e^{\mathrm{pos}}(x) + v_e^{\mathrm{neg}}(x) - v_o^{\mathrm{pos}}(x) - v_o^{\mathrm{neg}}(x) \\ i_1 &= i_e^{\mathrm{pos}}(x) - i_e^{\mathrm{neg}}(x) + i_o^{\mathrm{pos}}(x) - i_o^{\mathrm{neg}}(x) \\ i_2 &= i_e^{\mathrm{pos}}(x) - i_e^{\mathrm{neg}}(x) - i_o^{\mathrm{pos}}(x) + i_o^{\mathrm{neg}}(x) \end{aligned} \tag{12}$$

and (10) becomes

$$\begin{aligned} v_{\mathrm{dm}}(x) &= 2(v_o^{\mathrm{pos}}(x) + v_o^{\mathrm{neg}}(x)) \\ i_{\mathrm{dm}}(x) &= i_o^{\mathrm{pos}}(x) - i_o^{\mathrm{neg}}(x) = \frac{v_o^{\mathrm{pos}}(x) - v_o^{\mathrm{neg}}(x)}{Z_o} \\ v_{\mathrm{cm}}(x) &= v_e^{\mathrm{pos}}(x) + v_e^{\mathrm{neg}}(x) \\ i_{\mathrm{cm}}(x) &= 2(i_e^{\mathrm{pos}}(x) - i_e^{\mathrm{neg}}(x)) = 2\frac{v_e^{\mathrm{pos}}(x) - v_e^{\mathrm{neg}}(x)}{Z_e}. \end{aligned} \tag{13}$$

Note that, in general, $Z_o \neq Z_e$.

Characteristic impedances of each mode can be defined as the ratio of the voltage to current of the appropriate modes at any point, x, along the line. These impedances can be expressed in terms of the even and odd-mode (ground referenced) characteristic impedances

$$Z_{\mathrm{dm}} \equiv \frac{v_{\mathrm{dm}}^{\mathrm{pos}}(x)}{i_{\mathrm{dm}}^{\mathrm{pos}}(x)} = \frac{2v_o^{\mathrm{pos}}(x)}{v_o^{\mathrm{pos}}(x)/Z_o} = 2Z_o \tag{14}$$

$$Z_{\mathrm{cm}} \equiv \frac{v_{\mathrm{cm}}^{\mathrm{pos}}(x)}{i_{\mathrm{dm}}^{\mathrm{pos}}(x)} = \frac{v_e^{\mathrm{pos}}(x)}{(2v_e^{\mathrm{pos}}(x))/Z_e} = \frac{Z_e}{2}. \tag{15}$$