tunately, the generation and measurement of these modes of propagation is not easily achievable with standard vector network analyzers (VNA). However, under certain conditions, one can relate the total nodal waves (each representing two modes of propagation) to the desired differential and common-mode waves. These nodal waves are readily generated and measured with standard VNAs, and with consideration, the differential and common-mode waves, and hence the mixed-mode s-parameters, can be calculated. Therefore, the relationships between the normalized mixed-mode waves ($a_{\mathrm{dm1}}, b_{\mathrm{dm1}}, a_{\mathrm{cm1}}, b_{\mathrm{cm1}}$, etc.) and the nodal waves (a_1, b_1, a_2, b_2, etc.) will be derived, and the necessary conditions for these relationships to exist will be found.

If one is to make a general purpose RF measurement port, the values of characteristic port impedances must be chosen. It is useful to require the even and odd-mode characteristic impedances of the measurement system to be equal, thus reducing the number of different valued matched terminations required. In contrast, it is difficult to fabricate lumped termination standards for coupled lines where Z_e does not equal Z_o. If the characteristic impedances of the lines are defined to be equal (say, 50 Ω), then a further simplification of the above expressions can be accomplished with the substitution $Z_e = Z_o = Z_0$ where in the low-loss case $Z_0 \approx \mathrm{Re}\{Z_0\} \equiv R_0$.

By choosing equal even and odd-mode characteristic impedances, one is selecting a special case of coupled transmission line behavior, as described in (1). Enforcing equal even and odd-mode characteristic impedances is equivalent to the conditions of uncoupled transmission lines. As has been shown in the literature [7], the condition $Z_e = Z_o$ results in the mutual impedances and admittances being zero ($z_m = 0$, $y_m = 0$). Under these conditions, the describing differential equations of the transmission line system (1) clearly become uncoupled, resulting in two independent transmission line solutions. Although very specific, this is a valid solution to (1), and all results up to this point are also valid under the special case of equal even and odd-mode characteristic impedances. Therefore, we choose the reference lines of the mixed-mode s-parameters to be uncoupled transmission lines. The key to this choice is that these uncoupled reference lines can be easily interfaced with a coupled line system, as discussed below.

To interpret the meaning of uncoupled reference transmission lines, consider a system of transmission lines: one coupled pair, and one uncoupled pair connected in series with the coupled pair. If even and odd (or c and π) modes are both propagating (forward and reverse) on the coupled pair, then it can be shown that the waves propagating on each of the uncoupled transmission lines are linear combinations of the waves propagating on the coupled system (see Appendix). Furthermore, the differential and common-mode normalized waves of the coupled pair system can be reconstructed from the normalized waves at a point on the uncoupled line pairs (see Appendix). This point of reconstruction is arbitrary, and one may choose the point to be the interface between the coupled system and the uncoupled reference lines.

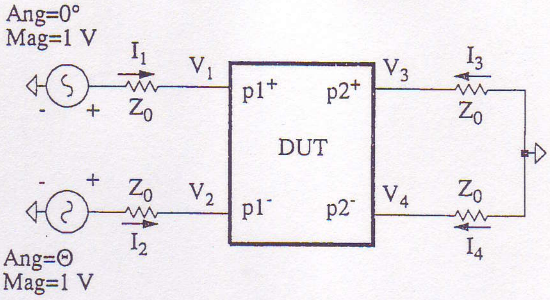

Fig. 4. Conceptual diagram of mixed-mode two-port measurement system.

Substituting $Z_e = Z_o = Z_0 \approx R_0$, the normalized nodal waves of the coupled lines at the interface are

$$a_i = \frac{1}{2\sqrt{R_0}}[v_i + i_i R_0]$$
$$b_i = \frac{1}{2\sqrt{R_0}}[v_i - i_i R_0] \tag{24}$$

where a_i and b_i are the normalized forward and reverse propagating nodal waves at node i, respectively, and $i \in \{1, 2, 3, 4\}$. These equations are applicable only in the case of low-loss lines, with equal even and odd mode characteristic impedance. By combining (12), (19), (20) and (24), it can be shown that the differential and common-mode waves a port 1 are

$$a_{\mathrm{dm}_1} = \frac{1}{\sqrt{2}}(a_1 - a_2)|_{x=0} \quad a_{\mathrm{cm}_1} = \frac{1}{\sqrt{2}}(a_1 + a_2)|_{x=0}$$
$$b_{\mathrm{dm}_1} = \frac{1}{\sqrt{2}}(b_1 - b_2)|_{x=0} \quad b_{\mathrm{cm}_1} = \frac{1}{\sqrt{2}}(b_1 + b_2)|_{x=0}. \tag{25}$$

Similarly, for port 2

$$a_{\mathrm{dm}_2} = \frac{1}{\sqrt{2}}(a_3 - a_4)|_{x=l} \quad a_{\mathrm{cm}_2} = \frac{1}{\sqrt{2}}(a_3 + a_4)|_{x=l}$$
$$b_{\mathrm{dm}_2} = \frac{1}{\sqrt{2}}(b_3 - b_4)|_{x=l} \quad b_{\mathrm{cm}_2} = \frac{1}{\sqrt{2}}(b_3 + b_4)|_{x=l}. \tag{26}$$

Equations (25) and (26) represent important relationships from which mixed-mode s-parameters can be determined with a practical measurement system. To understand the utility of the above relationships, consider Fig. 4, which is a conceptual model for a mixed-mode measurement system. By adjusting the phase difference, Θ, between the two sources to 0° or 180° one can determine the common-mode or differential-mode forward s-parameters, respectively. Conceptually, the measured quantities are the voltages and currents. These values can be related to the normalized nodal waves, a_1, b_1, a_2, b_2, etc., through the generalized definitions given in (24). From these nodal waves, the differential and common-mode normalized waves, and, hence, the mixed-mode s-parameters, can be calculated. Physically, the various ratios of nodal waves, a_1, b_1, a_2, b_2, etc., are measured, and from theses ratios the mixed-mode s-parameters are found.

The physical implementation of a mixed-mode s-parameter measurement system can be achieved with a modification of a standard VNA. The differential stimulus of a coupled two-port requires the input waves at the reference plane