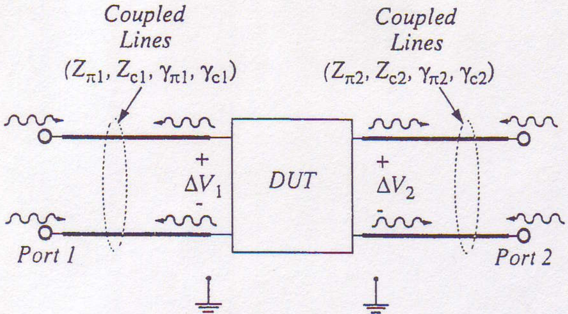

Fig. 1. Schematic of differential s-parameter measurement.

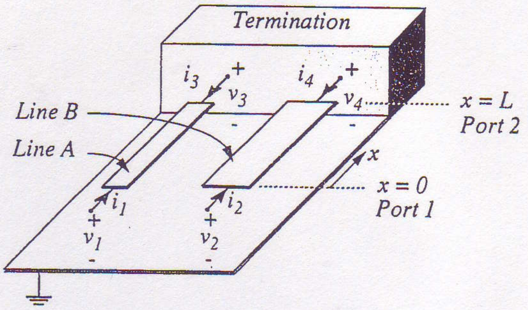

Fig. 2. Schematic of asymmetric coupled-pair transmission lines.

such a differential circuit is based on pairs of coupled transmission lines. A schematic of a typical two-port RF/microwave differential system is shown in Fig. 1. Essential features of a microwave differential circuit in Fig. 1 are the coupled pair transmission line input and output. It is conceptually beneficial to define a signal that propagates between the lines of the coupled-pair (as opposed to propagating between one line and ground). Such signals are known as differential signals, and can be described by a difference of voltage ($\Delta V_1 \neq 0, \Delta V_2 \neq 0$) and current flow between the individual lines in a pair. By such a definition, the signal is not referenced to a ground potential, but rather the signal on one line of the coupled pair is referenced to the other. Further, this differential signal should propagate in a TEM, or quasi-TEM, fashion with a well-defined characteristic impedance and propagation constant. Coupled line pairs, as in Fig. 1, allow propagating differential signals (the quantities of interest) to exist. The differential circuit discussion in this paper will be limited to the two-port case, but the generalized theory for n-port circuits can be readily derived from this work.

Most practical implementations of Fig. 1 will incorporate a ground plane, or some other global reference conductor, either intentionally or unintentionally. This ground plane allows another mode of propagation to exist, namely common-mode propagation. Conceptually, the common-mode wave applies equal signals with respect to ground at each of the individual lines in a coupled pair, such that the differential voltage is zero (i.e. $\Delta V_1 = \Delta V_2 = 0$). The ability of the microwave differential circuit to propagate both common-mode and differential-mode signals requires any complete theoretical treatment to include characterization of all simultaneously propagating modes. For convenience, the simultaneous propagation of two or more modes (namely, differential-mode, and common-mode) on a coupled transmission line will be referred to in this paper as mixed-mode propagation, from which mixed-mode s-parameters will be defined.

III. Mixed-Mode Power Waves and S-Parameters

To begin the presentation of mixed-mode s-parameters, a general asymmetric coupled transmission line pair over a ground plane will be analyzed. This analysis yields multiple propagating modes all referenced to ground. These modes will be used to express the desired differential signal between the lines of the coupled-pair, as well as the common signal referenced to ground. Fig. 2 is a diagram of such a coupled-pair transmission line, with all pertinent voltages and currents denoted. Also shown in Fig. 2 is a representation of a termination for the coupled-pair line. Later, this paper will use these lines as reference lines at the input and output of an arbitrary DUT. Subject to the simplifying assumptions, the mathematical results of this paper are applicable to any pair of conductors with a nearby ground conductor.

Referring again to Fig. 2, the behavior of the coupled-line pair can be described by [6]

$$\begin{aligned} -\frac{dv_1}{dx} &= z_1 i_1 + z_m i_2 \\ -\frac{dv_2}{dx} &= z_2 i_2 + z_m i_1 \\ -\frac{di_1}{dx} &= y_1 v_1 + y_m v_2 \\ -\frac{di_2}{dx} &= y_2 v_2 + y_m v_1 \end{aligned} \tag{1}$$

where z_1 and z_2 are self-impedances per unit length, y_1 and y_2 are admittances per unit length, and z_m and y_m are mutual impedance and admittance per unit length, respectively. Also, a harmonic time dependance (i.e. $e^{j\omega t}$) is assumed.

The solution to the set of (1) as published by Tripathi [6] is given as

$$\begin{aligned} v_1 &= A_1 e^{-\gamma_c x} + A_2 e^{\gamma_c x} + A_3 e^{-\gamma_\pi x} + A_4 e^{\gamma_\pi x} \\ v_2 &= A_1 R_c e^{-\gamma_c x} + A_2 R_c e^{\gamma_c x} + A_3 R_\pi e^{-\gamma_\pi x} + A_4 R_\pi e^{\gamma_\pi x} \\ i_1 &= \frac{A_1}{Z_{c1}} e^{-\gamma_c x} - \frac{A_2}{Z_{c2}} e^{\gamma_c x} + \frac{A_3}{Z_{\pi 1}} e^{-\gamma_\pi x} - \frac{A_4}{Z_{\pi 2}} e^{\gamma_\pi x} \\ i_2 &= \frac{A_1 R_c}{Z_{c1}} e^{-\gamma_c x} - \frac{A_2 R_c}{Z_{c2}} e^{\gamma_c x} + \frac{A_3 R_\pi}{Z_{\pi 1}} e^{-\gamma_\pi x} - \frac{A_4 R_\pi}{Z_{\pi 2}} e^{\gamma_\pi x} \end{aligned} \tag{2}$$

where A_1, and A_3 represent the phasor coefficients for the forward (positive x) propagating c and π-modes, respectively, and A_2, and A_4 represent the phasor coefficients for the reverse (negative x) propagating c and π-modes, respectively. The characteristic impedance of the c-modes are represented by Z_{c1} and Z_{c2} for lines A and B, respectively, and the characteristic impedance of the π-modes are represented by $Z_{\pi 1}$ and $Z_{\pi 2}$ for lines A and B, respectively. Additionally, $R_c = v_2/v_1$ for $\gamma = \pm\gamma_c$, $R_\pi = v_2/v_1$ for $\gamma = \pm\gamma_\pi$, and

$$\begin{aligned} \gamma_{c,\pi}^2 = \frac{y_1 z_1 + y_2 z_2}{2} + y_m z_m \pm \frac{1}{2}[(y_1 z_1 - y_2 z_2)^2 \\ + 4(z_1 y_m + y_2 z_m)(z_2 y_m + y_1 z_m)]^{1/2}. \end{aligned} \tag{3}$$